Definition 1. A **set** is an unordered collection of mathematical objects (often numbers, but a set may contain other things as well, even other sets). The objects in the set are called **elements** regardless of what they are (numbers, vectors, other sets, etc.). If S is a set and a is an element of S, we write $a \in S$. If a is not an element of S, we write $a \notin S$.

Duplicate elements of a set are ignored, i.e. there is no concept of an element belonging to a set more than one time. Everything is either in or out.

Definition 2. The most basic way to describe a set is by listing all of its elements, separated by commas. When we do this, we use brackets { and } to denote the start and end of the list of elements.

Example 1. $S = \{1, 2, 5\}$ is a set with three elements. $\{1, 5, 2\}$ is the same set, because the order of elements doesn't matter. Also, $\{1, 1, 2, 5\}$ is the same set, because the duplicate 1 is irrelevant. With this set, we can say that

$$1 \in S \qquad 2 \in S \qquad 3 \notin S \qquad 4 \notin S \qquad 5 \in S$$

Example 2. It is possible for a set to have no elements. $A = \{\}$ is a set. For example, $1 \notin A$. Since any two sets with no elements are the same, we speak of "the" empty set. The symbol $\varnothing$ is used for this set.

Definition 3. Another way to describe a set is by listing some elements which intend to establish some pattern, then writing ellipses $(\ldots)$.

Example 3. The set of positive integers can be written as $\{1, 2, 3, \ldots\}$. The set of all integers can be described using $\{\ldots, -2, -1, 0, 1, 2, \ldots\}$.

Definition 4. A much more precise and powerful way to describe sets is with **set-builder notation**. One form of set-builder notation looks like

$$A = \{f(x) : x \in B\}$$

where B is some previously known set, and $f(x)$ is some expression involving x. This form of set builder notation describes the set A as a list of elements of the form $f(x)$, where x ranges over a set B.

Example 4. We can describe the set of even integers as

$$A = \{2x : x \in \mathbb{Z}\}$$

This says that for any integer x, $2x$ is an element of A. So since $1 \in \mathbb{Z}$, $2(1) \in A$. Since $2 \in \mathbb{Z}, 2(2) = 4 \in A$. We could alternately describe the same set A using listing and ellipses as

$$A = \{\ldots, -6, -4, -2, 0, 2, 4, 6, \ldots\}$$

Question 1. List 5 different elements of the set $B = \{x^2 + 1 : x \in \mathbb{Z}\}$.

Question 2. Let $C = \{4n + 2 : n \in \mathbb{Z}\}$. Is $99 \in C$? Is $100 \in C$? What is the smallest integer greater than 100 that is an element of C?

Question 3. Let D be the set of integers which are multiples of 3. Describe D using listing with ellipses, then also describe D using set-builder notation.

Question 4. Let E be the set of positive real numbers. Describe E using set-builder notation.