This document expands on the syllabus description of Unit 1 – Notation and sets. It is provided as a study reference for you to prepare for quizzes on this unit and the final exam.

**Quiz dates for unit 1**

| | |
|---|---|
| Quiz 1A | Wednesday, February 4 |
| Quiz 1B | Friday, February 13 |

**Syllabus description of unit 1**

I can distinguish matematical data types: boolean, number, tuple/vector, set, statement, and variable statement. I can use and read these symbols:

$\emptyset, \mathbb{N}, \mathbb{Z}, \mathbb{Q}, \mathbb{R}, \mathbb{C}, \in, \notin, \subseteq, \nsubseteq, \subsetneq, \cup, \cap, \langle a, b\rangle, \{a, b\}, (a, b), [a, b], |x|, \lfloor x\rfloor, \lceil x\rceil, \vee, \wedge, \oplus, \neg, \implies, \iff, \forall, \exists$

**Core content**

1. Identify the types of objects described using the various delimiters: $||$, $\lceil\rceil$, $\lfloor\rfloor$, $\langle\rangle$, $\{\}$, $()$, and $[]$. For example, what is the type of each object below?

$$1 \qquad |1| \qquad \lceil 1\rceil \qquad \lfloor 1\rfloor \qquad \langle 1,2\rangle \qquad \{1,2\} \qquad [1,2] \qquad (1,2)$$

2. Distinguish between $\in, \notin, \subseteq, \nsubseteq$ and $\subsetneq$, particularly when it comes to identifying whether a usage is a type error. Label each of the following as true, false, or type error.

| | | | |
|---|---|---|---|
| $1 \in \langle 1,2\rangle$ | $1 \in \{1,2\}$ | $1 \in (1,2)$ | $1 \in [1,2]$ |
| $1 \notin \langle 1,2\rangle$ | $1 \notin \{1,2\}$ | $1 \notin (1,2)$ | $1 \notin [1,2]$ |
| $1 \subseteq \langle 1,2\rangle$ | $1 \subseteq \{1,2\}$ | $1 \subseteq (1,2)$ | $1 \subseteq [1,2]$ |
| $\{1\} \in \langle 1,2\rangle$ | $\{1\} \in \{1,2\}$ | $\{1\} \in (1,2)$ | $\{1\} \in [1,2]$ |
| $\{1\} \subseteq \langle 1,2\rangle$ | $\{1\} \subseteq \{1,2\}$ | $\{1\} \subseteq (1,2)$ | $\{1\} \subseteq [1,2]$ |

| | | | |
|---|---|---|---|
| $0 \in [0,1]$ | $0 \subseteq [0,1]$ | $0 \nsubseteq [0,1]$ | $0 \subsetneq [0,1]$ |
| $\emptyset \in [0,1]$ | $\emptyset \subseteq [0,1]$ | $\emptyset \nsubseteq [0,1]$ | $\emptyset \subsetneq [0,1]$ |
| $[0,1] \in [0,1]$ | $[0,1] \subseteq [0,1]$ | $[0,1] \nsubseteq [0,1]$ | $[0,1] \subsetneq [0,1]$ |
| $[0,2] \in [0,1]$ | $[0,2] \subseteq [0,1]$ | $[0,2] \nsubseteq [0,1]$ | $[0,2] \subsetneq [0,1]$ |

3. Translate a statement written using symbolic logic symbols $\vee, \wedge, \oplus, \neg, \implies, \iff$ into an English sentence. For example, translate these statements:

$$\Big(2 \in [0,10]\Big) \wedge \Big(\emptyset \subseteq [0,10]\Big)$$
$$n \in \mathbb{N} \implies n \geq 0$$

**Other kinds of questions**

1. Distinguish between $\mathbb{N}, \mathbb{Z}, \mathbb{Q}, \mathbb{R}$, and $\mathbb{C}$ by giving examples of numbers that belong to one but not the other. For each pair below, give an example of a number that belongs to the bigger set but not the smaller set.

$$\mathbb{N} \text{ and } \mathbb{Q} \qquad\qquad \mathbb{R} \text{ and } \mathbb{Q}$$

2. Distinguish between $|x|$, $\lceil x \rceil$ , and $\lfloor x \rfloor$. For example, compute each of the following.

$$|-8.5| \qquad \lceil -8.5 \rceil \qquad \lfloor -8.5 \rfloor \qquad |\pi| \qquad \lceil \pi \rceil \qquad \lfloor \pi \rfloor$$

3. Translate a statement written using symbolic logic symbols $\vee, \wedge, \oplus, \neg, \implies, \iff, \forall, \exists$ into an English sentence. For example, translate these statements:

$$\begin{aligned}
&\forall k \in \mathbb{Z}, k \in \mathbb{Q} \\
&\exists x \in \mathbb{R}, |x| < 0 \\
&\neg\Big(\exists x \in \mathbb{R}, |x| < 0\Big) \\
&\exists x \in \mathbb{R}, \neg\Big(|x| < 0\Big)
\end{aligned}$$

4. Describe the differences between $0$ and $\{0\}$ and $\emptyset$.

5. Describe the differences between $(4, 7)$ and $[4, 7]$ and $\{4, 7\}$ and $\langle 4, 7 \rangle$.

6. Translate each of the following variable statements from symbolic logic into an English sentence. For the purposes of these statements, $k \in \mathbb{Z}$. Then give an example of a specific value of $k$ for which one statement is true but the other is false.

$$\begin{aligned}
&(k < 20) \vee (k^2 > 100) \\
&(k < 20) \oplus (k^2 > 100)
\end{aligned}$$