1. How many vectors can be in a basis of $\mathbb{R}^m$? Is there a maximum, a minimum, a range?

2. Let $A = \begin{pmatrix} 1 & 5 & -2 & 3 & 0 \\ 3 & 2 & 0 & -1 & 1 \\ 5 & 12 & -4 & 5 & 1 \end{pmatrix}$

   (a) Let $W = \mathrm{Col}(A)$. What vector space is W a subspace of?

   (b) Find a basis for W, using only columns of A.

3. Let $W = \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} \in \mathbb{R}^3 : x + y + z = 0 \right\}$ and let $\vec{u}_1 = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$, $\vec{u}_2 = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$, and $\vec{u}_3 = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$.

   (a) Verify that $\vec{u}_1, \vec{u}_2, \vec{u}_3 \in W$.

   (b) Show that $(\vec{u}_1, \vec{u}_2, \vec{u}_3)$ is linearly dependent.

   (c) It is true that $\operatorname{span}(\vec{u}_1, \vec{u}_2, \vec{u}_3) = W$ (you do not have to prove this). Therefore, W is equal to the column space of the matrix

$$A = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & -1 \\ -1 & -1 & 0 \end{pmatrix}$$

   Using this fact, find a basis of W.

4. Find a basis for the column space of the matrix

$$A = \begin{pmatrix} 0 & 1 & 1 & 2 & 1 \\ 1 & 1 & 2 & 3 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 2 & 3 & 1 \end{pmatrix}$$