

1. Let $f : R \rightarrow S$ be a ring homomorphism.

(a) Given the definition of the kernel of f .

(a) Give the definition of the image of f .

2. Let $R = \mathbb{Z}$ and $S = \mathbb{Z}/8\mathbb{Z}$ and let $f : R \rightarrow S$ be the function $f(x) = [2x]$.

(a) Describe the kernel of f .

(a) Describe the image of f .

3. Let $R = \mathbb{R}[x]$ and let $I = (x)$ be the principal ideal generated by x .

(a) List three different elements of I .

(a) List three different elements of R that are not elements of I .

4. Which of the following are ideals of $\mathbb{Z}$?

$$3\mathbb{Z} = \{\dots, -6, -3, 0, 3, 6, \dots\}$$

$$1 + 3\mathbb{Z} = \{\dots, -5, -2, 1, 4, 7, \dots\}$$

$$2 + 3\mathbb{Z} = \{\dots, -4, -1, 2, 5, 8, \dots\}$$

$$3 + 3\mathbb{Z} = \{\dots, -3, 0, 3, 6, 9, \dots\}$$