

Name: _____

1. Let $G = \mathbb{Z}/3\mathbb{Z} \times \mathbb{Z}/3\mathbb{Z} = \{(a, b) : a \in \mathbb{Z}/3\mathbb{Z} \text{ and } b \in \mathbb{Z}/3\mathbb{Z}\}$.

(a) Is G abelian?

(b) What is the inverse of $(2, 2)$?

(c) Describe the cyclic subgroup generated by $(1, 2)$.

(d) Find the order of each element of G (there are 9 elements).

(e) Is G cyclic? Explain the relevance of part (d) to this question.

2. Let $G = \mathbb{C} = \{a + bi : a, b \in \mathbb{R}\}$, with the group operation being addition. Let $H = \mathbb{Z}[i] = \{c + di : c, d \in \mathbb{Z}\}$. Prove that H is a subgroup of G .

3. Let G be a group and let $g, h \in G$. Prove that $(gh)^{-1} = h^{-1}g^{-1}$.