Math 319 – Algebraic Structures 1 – Fall 2024

Course meetings
10-10:50am MWF
Birdsong Hall 101

Instructor
Joshua Ruiter
jruiter@vwu.edu
Office: Eggleston Commons 110
Office hours: 10-12 Tue/Th

What is this course? Addition and multiplication have properties like "associativity" and "commutativity." In abstract algebra we generalize and study properties like these for other operations, such as matrix multiplication or function composition. What properties do other operations have? Given a list of properties, what possible operations can satisfy them? What are the consequences of different combinations of properties?

Our central two examples throughout the course will be the world of integers mod n, and the world of polynomials. Both of these are places where we can add and multiply, but not everything works exactly like "normal" additional and multiplication.

Course objectives: By the end of the semester, a successful student will be able to reason about equivalence relations, rings, fields, ring homomorphisms and isomorphisms, quotient rings, and polynomial rings. "Reason about" includes articulating formal definitions and axioms, as well as having a repertoire of useful examples and counterexamples. A successful student will also improve in their ability to communicate mathematics effectively, both verbally and through written proofs.

Textbook: *Algebra: Notes from the Underground* by Paolo Aluffi.

Grades: Final grades will be computed via weighted average. The cutoffs for letter grades are shown in the table.

Attendance	10%
Homework	25%
Quizzes	25%
Exams (3)	10% each
Final exam	10%

0	60	63	67	70	73	77	80	83	87	90	93
F	D−	D	D+	C−	C	C+	B−	B	B+	A−	A

Attendance: Attendance will be taken every day. Missing class for any reason is considered an absence, regardless of circumstances. Excessive lateness or non-participation may be counted as an absence. Anyone with more than 6 absences will automatically fail the course.

Participation includes working on problems given during class, particularly those on passed our worksheets. Non-participation will be counted as an absence for that day. Being late, leaving the classroom during class, or otherwise being virtually abset (i.e. using a phone, laptop in a way not relevant to class) will be counted as an absence.

Quizzes: There will be a quiz on each Friday without an exam. Quizzes will ask about definitions, examples, and computational questions.

Exams: Three in class exams on Friday, September 20; Friday, October 25; and Friday, November 15, and a cumulative final exam on Friday, December 13 at 11:30am-2pm. Exams will ask you to write proofs. If you will miss an exam, let me know in advance. If the final exam is higher than your lowest exam score, I will replace your lowest exam score with your score on the final.

Homework: There will be a problem set due each Wednesday at the start of class. You are welcome to work with classmates on homework, but you must write down your own solutions.

Grading and resubmission: You can resubmit problems the next week for full credit. I will return homework quickly to give you sufficient time for revisions. To resubmit a problem, include it with the next homework set and attach the previous attempt on the same problem.

How to go about doing homework:

- Assume you should always justify your answer, even if the problem doesn't say that explicitly. For example, if a problem starts with "Find a blah..." or "Give an example of blah..." there is an implicit "... and explain why."
- All solutions should be written in complete sentences and in paragraph form. For example, the following is **not** a complete sentence.

$$\lim_{n\to\infty} \frac{n+n^2}{3n^2} = \lim_{n\to\infty} \frac{n}{3n^2} + \lim_{n\to\infty} \frac{n^2}{3n^2} = \lim_{n\to\infty} \frac{1}{3n} \overset{0}{\nearrow} + \lim_{n\to\infty} \frac{1}{3} \overset{\frac{1}{3}}{\nearrow} = 0 + \frac{1}{3} = \frac{1}{3}$$

Formatting this sequence of reasoning in complete sentences would look more like

> The limit $\lim_{n\to\infty} \frac{n}{3n^2} = \lim_{n\to\infty} \frac{1}{3n}$ converges to zero and the limit $\lim_{n\to\infty} \frac{n^2}{3n^2} = \lim_{n\to\infty} \frac{1}{3}$ converges to $\frac{1}{3}$, so we can apply the fact that the limit of a sum is a sum of limits and conclude that the limit $\lim_{n\to\infty} \frac{n+n^2}{3n^2} = \lim_{n\to\infty} \left(\frac{n}{3n^2} + \frac{n^2}{3n^2} \right)$ converges to $\frac{1}{3}$.

- When writing solutions, don't think of writing them for my benefit or to convince me of something. Imagine you are trying to convince a classmate or yourself.
- You don't need to justify results we proved in class.
- If you are stuck, try making sure you know what all the mathematical words and symbols refer to, refresh yourself on any definitions, or try doing a smaller example.

Disability services: Virginia Wesleyan University is committed to giving all students the opportunity of academic success. If you are a student who is requesting accommodations based on the academic impact of a disability, speak to me about your accommodations letter and your specific needs. If you do not have an accommodation letter for this course, you will need to visit or call for an appointment with the Executive Director of Academic Support Services, Crit Muniz at (757) 233-8898 or by email at disabilityservices@vwu.edu to coordinate reasonable accommodations. His office is in Clarke 204.

Key dates

September 20	Exam 1
October 25	Exam 2
October 14	No class, Fall break
November 22	Exam 3
November 27, 29	No class, Thanksgiving break
Decmeber 9	Last day of class
December 13	Final exam

Schedule

Day	Section	Topic	Assessment
Week 1			
August 28	1.1	Well-ordering and induction	
August 30	1.2	Integer division with remainder	
Week 2			
September 2	1.2, 1.3	More on integer division, GCD	
September 4	1.3	Greatest common divisor	HW1
September 6	1.4	Fundamental theorem of arithmetic	Quiz 1
Week 3			
September 9	1.4, 2.1	FTA, equivalence relations	
September 11	2.2	Arithmetic mod n	HW2
September 13	2.3	Arithmetic mod n	Quiz 2
Week 4			
September 16	2.3, 2.4	Airthmetic mod n	
September 18	2.4	Arithmetic mod n	HW3
September 20			Exam 1
Week 5			
September 23	3.1	Rings - definition and examples	
September 25	3.2	Rings - basic properties	HW4
September 27	3.2	Rings	Quiz 3
Week 6			
September 30	3.2	Rings	
October 2	3.3	Rings	HW5
October 4	4.1	Cartesian products	Quiz 4
Week 7			
October 7	4.1, 4.2	Cartesian products, subrings	
October 9	4.2	Subrings	HW6
October 11	4.3	Ring homomorphisms	Quiz 5

Week 8			
October 14		No class, fall break	
October 16	4.3, 4.4	Ring homomorphisms and isomorphisms	
October 18	4.4	Ring isomorphisms	Quiz 6

Week 9			
October 21	5.1, 5.2	Kernels and ideals	
October 23	5.1, 5.2	Kernels and ideals	HW8
October 25			Exam 2

Week 10			
October 28	5.3	Cosets of ideals	
October 30	5.3	Cosets of ideals	HW9
November 1	5.3	Quotient rings	Quiz 7

Week 11			
November 4	5.3	Quotient rings	
November 6	5.4	First isomorphism theorem	HW10
November 8	5.5	First isomorphism theorem	Quiz 8

Week 12			
November 11	5.6	Second isomorphism theorem	
November 13	5.7	Third isomorphism theorem	HW11
November 15			Exam 3

Week 13			
November 18	7.2	Polynomial rings	
November 20	7.3	Polynomial rings	HW12
November 22	7.4	Polynomial rings	Quiz 9

Week 14			
November 25	7.4	Polynomial rings	
November 27		No class, Thanksgiving break	
November 29		No class, Thanksgiving break	

Week 15			
December 2	7.5	Polynomial rings	
December 4		Polynomial rings	HW13
December 6		Review	Quiz 10

Week 16			
December 9		Review, last day of class	

Exam week			
December 13		11:30am-2pm Final exam	