Math 321
Foundations of Abstract Algebra

Course meetings
1:00-1:50pm MWF
2245 Noyce

Instructor
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Office hours: 11am-Noon, 1-2pm Tues, Thu

What is this course? Addition and multiplication have properties like "associativity" and "commutativity." In abstract algebra we generalize and study properties like these for other operations, such as matrix multiplication or function composition. What properties do other operations have? Given a list of properties, what possible operations can satisfy them? What are the consequences of different combinations of properties? In the first half of the course, we will study sets with only a single operation (groups), and in the second half we will study sets with two operations (rings).

Course objectives: By the end of the semester, a successful student will be able to

- Articulate definitions, examples, and counterexamples of algebraic structures such as groups, rings, and fields.
- Prove theorems about algebraic structures.
- Communicate mathematics effectively in writing, primarily through writing readable, correct, and rigorous proofs.

Course policies

Communication: You will need to check Blackboard regularly for participation assignments. If you need to get in touch with me outside class, come to my office hours or send me an email. If my office hours conflict with your schedule, send me an email and we can find another time to meet.

Workload: I expect most of the time you spend on this class outside class to be on homework. If you are having to spend more than 9 hours a week outside class, please let me know and we can talk about reducing the workload.

Accommodations: I encourage students with documented disabilities to discuss appropriate accommodations with me. You will also need to have a conversation with, and provide documentation of your disability to, the Coordinator of Disability Resources, John Hirschman, located on the third floor of Goodnow Hall (x3089).

Religious holidays: Let me know early in the course if you will miss class because of a religious holiday, especially if it is a HW due date or exam, so we can plan together how you can still fulfil the requirements of the course.

Textbook: *Contemporary Abstract Algebra (8th edition)*, Joseph A. Gallian. Having a copy of the textbook will be important for the daily participation assignments, as well as sourcing homework problems. You can accomplish this with a paper or electronic copy.

Grades

Final grades will be computed via weighted average.

Participation	10%
Homework	35%
Midterm 1	15%
Midterm 2	15%
Final exam	25%

$[94, 100]$	A
$[90, 94)$	A-
$[87, 90)$	B+
$[84, 87)$	B
$[80, 84)$	B-
$[77, 80)$	C+
$[70, 77)$	C
$[60, 70)$	D
$[0, 60)$	F

If your final exam score exceeds your lowest midterm score, the final exam score will substitute for that exam. In that case, the calculation would be

Participation	10%
Homework	35%
Higher midterm	15%
Final exam	40%

Participation: Each class day will have an associated short reading and short problem which you turn in at the start of class. The short problem will often take the form, "Give an example of A and an example of (not A)." These will usually **not** require a substantial argument or write-up. You can miss up to two of these without penalty.

Homework: Problem sets will be due on Fridays at the start of class. See below for more info.

Exams: Two in class midterms on Friday, October 7 and Monday, November 21, and a cumulative final exam on Wednesday, December 14 at 2pm-5pm. You will be able to bring notes.

Seminars: Each exam will have an associated small extra credit opportunity, involving the weekly Thursday math and statistics seminar. Attend a seminar, write a 1-2 paragraph summary of the talk you saw, and turn that in any time before the exam. The exact amount of points is TBD depending on the total points of the exam.

Academic honesty: Make sure you are familiar with the college's guidelines for academic honesty which you can find here: `http://catalog.grinnell.edu/content.php?catoid=12&navoid=2537#Honesty_in_Academic_Work`. There are very serious consequences if you are found to be in violation of one of these policies. A typical first offense is a zero on the particular assignment and your final grade in the course is dropped a full letter grade.

You are welcome to work with classmates on homework, but you must write down your own solution.

- 👍 Discussing a problem with classmates, one of you has a key insight and shares it with everyone, then everyone writes their own solutions.
- 👍 You and a classmate worked together on a problem and look at and discuss your written solutions together.

👍 Someone solves a problem on their own, then gives you a hint.

👎 Someone solves a problem on their own, then tells you key pieces of the proof.

👎 You look up a solution on the internet.

In all of the 👍 examples, you should indicate on your homework that someone helped, like "Joshua helped me on this problem."

Homework

Grading and resubmission: Weekly homework sets will have around 7 problems, graded 0/1/2.

2 Correct solution with sufficient justifications, only very minor errors.

1 Something on the right track, but lacks justification or skipped a step.

0 Something is unclear or there is a logical mistake.

You can resubmit any problem up to two weeks later, and earn full credit. For example, a problem due on September 2 could be resubmitted on September 9 and again on September 16, but not after that. I will try to grade and return homeworks quickly to give you sufficient time for revisions. To resubmit a problem, include it with the next homework set and attach previous attempts on the same problem.

Late work: Over the course of the semester, each student has two chances to submit a homework late with no penalty. It must be submitted at the start of class on the Monday following the Friday it was due. If you intend to do this, you must tell me before Friday class. No other late work will be accepted.

How to go about doing homework:

- Assume you should always justify your answer, even if the problem doesn't say that explicitly. For example, if a problem starts with "Find a blah..." or "Give an example of blah..." there is an implicit "... and explain why."
- All solutions should be written in complete sentences and in paragraph form. For example, the following is **not** a complete sentence.

$$\lim_{n\to\infty} \frac{n+n^2}{3n^2} = \lim_{n\to\infty} \frac{n}{3n^2} + \lim_{n\to\infty} \frac{n^2}{3n^2} = \lim_{n\to\infty} \frac{1}{3n} \overset{0}{\nearrow} + \lim_{n\to\infty} \frac{1}{3} \overset{\frac{1}{3}}{\nearrow} = 0 + \frac{1}{3} = \frac{1}{3}$$

Formatting this sequence of reasoning in complete sentences would look more like

> The limit $\lim_{n\to\infty} \frac{n}{3n^2} = \lim_{n\to\infty} \frac{1}{3n}$ converges to zero and the limit $\lim_{n\to\infty} \frac{n^2}{3n^2} = \lim_{n\to\infty} \frac{1}{3}$ converges to $\frac{1}{3}$, so we can apply the fact that the limit of a sum is a sum of limits and conclude that the limit $\lim_{n\to\infty} \frac{n+n^2}{3n^2} = \lim_{n\to\infty} \left(\frac{n}{3n^2} + \frac{n^2}{3n^2} \right)$ converges to $\frac{1}{3}$.

- When writing solutions, don't think of writing them for my benefit or to convince me of something. Imagine you are trying to convince a classmate or yourself.
- You don't need to justify results we proved in class.
- If you are stuck, try making sure you know what all the mathematical words and symbols refer to, refresh yourself on any definitions, or try doing a smaller example.

LaTeX: You can submit homework handwritten or typed in LaTeX. Either can be turned in on paper or electronically via Gradescope. Handwritten work (paper or electronic) is due by the start of class on Fridays. Work done in LaTeX can be turned in by 5pm on the same Friday.

If you want any help with LaTeX, I am happy to get you started and provide some templates. It can be very difficult at first, but it is an extremely rewarding and useful skill. If you can envision yourself going to graduate school in a STEM field, you might need to learn it eventually so now is a great time.

Schedule

Day	Textbook	Topics	Homework
Week 0			
Aug 26	Ch0	Equivalence relations, modular arithmetic	
Week 1			
Aug 29	Ch1	Symmetries of a square, dihedral groups	Introductions
Aug 31	Ch2	Group axioms	
Sept 2	Ch2	Cancellation and other properties of groups	HW1 due
Week 2			
Sept 5	Ch3	Orders of groups and elements, subgroups	
Sept 7	Ch3	Cyclic subgroups	
Sept 9	Ch4	Cylic groups	HW2 due
Week 3			
Sept 12	Ch4	Subgroups of cyclic groups	
Sept 14	Ch5	Permutation groups, symmetric groups	
Sept 16	Ch5	Even and odd permutations, alternating group	HW3 due
Week 4			
Sept 19	Ch6	Group isomorphisms	
Sept 21	Ch6	Cayley's theorem	
Sept 23	Ch7	Cosets	HW4 due
Week 5			
Sept 26	Ch7	Lagrange's theorem	
Sept 28	Ch8	Direct products of groups	
Sept 30	Ch8	Products of cyclic groups	HW5 due
Week 6			
Oct 3	Ch9	Normal subgroups	
Oct 5	Ch9	Quotient groups	
Oct 7	Ch9	Midterm 1	(no HW)
Week 7			
Oct 10	Ch10	Group homomorphisms	
Oct 12	Ch10	Kernels, first isomorphism theorem	
Oct 14	Ch11	Classification of finite abelian groups	HW7 due
Oct 17-21	Break		

Day	Textbook	Topics	Homework
Week 8			
Oct 24	Ch12	Ring axioms	
Oct 26	Ch12	Subrings	
Oct 28	Ch13	Zero divisors and integral domains	HW8 due
Week 9			
Oct 31	Ch13	Finite integral domains and fields	
Nov 2	Ch14	Ideals and quotient rings	
Nov 4	Ch14	Prime ideals and maximal ideals	HW9 due
Week 10			
Nov 7	Ch15	Ring homomorphisms	
Nov 9	Ch15	Properties of ring homomorphisms	
Nov 11	Ch15	Prime subfields, field of quotients	HW10 due
Week 11			
Nov 14	Ch16	Polynomial rings	
Nov 16	Ch16	Division algorithm for polynomials	
Nov 18	Ch16	Principal ideal domains	HW11 due
Week 12			
Nov 21	Ch17	Midterm 2	
Nov 23	Ch17	Irreducibillity of polynomials	
Nov 25	Break		(no HW)
Week 13			
Nov 28	Ch17	Irreducibility tests (Eisenstein's criterion)	
Nov 30	Ch17	Ideals generated by irreducible polynomials	
Dec 2	Ch18	Unique factorization in $\mathbb{Z}[x]$, FLT	HW13 due
Week 14			
Dec 5	Ch18	Irreducible and prime elements	
Dec 7	Ch18	Unique factorization domains	
Dec 9	Ch18	Euclidean domains	HW14 due
Finals			
Dec 14	2pm-5pm	Final exam	